Formelblad matematik 4

Algebra

Regler

$$(a+b)^2 = a^2 + 2ab + b^2 \qquad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^2 = a^2 - 2ab + b^2 \qquad (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a+b)(a-b) = a^2 - b^2$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Andragradsekvationer

$$x^2 + px + q = 0 \qquad ax^2 + bx + c = 0$$

$$x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \qquad x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Aritmetik

Prefix

T	G	M	k	h	d	c	m	μ	n	p
tera	giga	mega	kilo	hekto	deci	centi	milli	mikro	nano	piko
10^{12}	10^{9}	10^{6}	10^{3}	10^{2}	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}

Potenser

$$a^x a^y = a^{x+y} \qquad \frac{a^x}{a^y} = a^{x-y} \qquad (a^x)^y = a^{xy} \qquad a^{-x} = \frac{1}{a^x}$$

$$a^x b^x = (ab)^x \qquad \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x \qquad a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^0 = 1$$

Geometrisk summa

$$a + ak + ak^2 + \ \ldots\ + ak^{n-1} = \frac{a(k^n - 1)}{k - 1} \quad \text{där } k \neq 1$$

Logaritmer

$$y = 10^x \Leftrightarrow x = \lg y \qquad y = \mathrm{e}^x \Leftrightarrow x = \ln y$$

$$\lg x + \lg y = \lg xy \qquad \lg x - \lg y = \lg \frac{x}{y} \qquad \lg x^p = p \cdot \lg x$$

Absolutbelopp

$$|a| = \begin{cases} a & \text{om } a \geq 0 \\ -a & \text{om } a < 0 \end{cases}$$

Funktioner och samband

Räta linjen

$y = kx + m$ $\qquad k = \dfrac{y_2 - y_1}{x_2 - x_1}$

$k_1 \cdot k_2 = -1$, villkor för vinkelräta linjer

$ax + by + c = 0$, där inte både a och b är noll

Andragradsfunktioner

$y = ax^2 + bx + c$ $\qquad a \neq 0$

Potensfunktioner

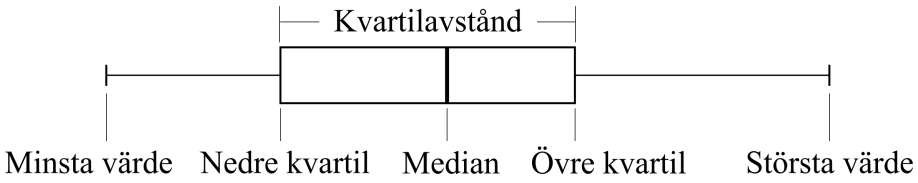
$y = C \cdot x^a$

Exponentialfunktioner

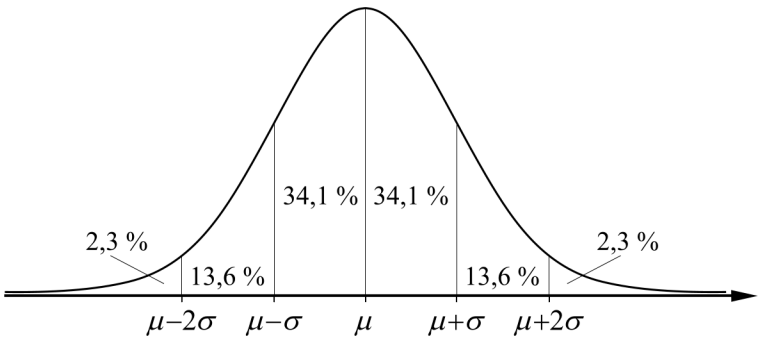
$y = C \cdot a^x$ $\qquad a > 0$ och $a \neq 1$

Statistik och sannolikhet

Lådagram



Normalfördelning



Täthetsfunktion för normalfördelning

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Differential- och integralkalkyl

Derivatans definition $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

Derivator

Funktion	Derivata
x^n där n är ett reellt tal	nx^{n-1}
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\ln x \quad (x > 0)$	$\frac{1}{x}$
$a^x \quad (a > 0)$	$a^x \ln a$
e^x	e^x
e^{kx}	$k \cdot \mathrm{e}^{kx}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$1 + \tan^2 x = \frac{1}{\cos^2 x}$
$k \cdot f(x)$	$k \cdot f'(x)$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$\frac{f(x)}{g(x)} \quad (g(x) \neq 0)$	$\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$

Kedjeregeln Om $y = f(z)$ och $z = g(x)$ är två deriverbara funktioner så gäller för $y = f(g(x))$ att

$y' = f'(g(x)) \cdot g'(x)$ eller $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}z} \cdot \frac{\mathrm{d}z}{\mathrm{d}x}$

Integralkalkylens fundamentalsats

$$\int_a^b f(x)\,\mathrm{d}x = \left[F(x)\right]_a^b = F(b) - F(a) \quad \text{där } F'(x) = f(x)$$

Primitiva funktioner

Funktion	Primitiva funktioner
k	$kx + C$
$x^n \quad (n \neq -1)$	$\dfrac{x^{n+1}}{n+1} + C$
$\dfrac{1}{x}$	$\ln x + C \quad (x > 0)$
$a^x \quad (a > 0,\ a \neq 1)$	$\dfrac{a^x}{\ln a} + C$
e^x	$\mathrm{e}^x + C$
e^{kx}	$\dfrac{\mathrm{e}^{kx}}{k} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$

Rotationsvolym

$$V = \pi \cdot \int_a^b y^2\,\mathrm{d}x \quad \text{rotation runt } x\text{-axeln}$$

$$V = \pi \cdot \int_a^b x^2\,\mathrm{d}y \quad \text{rotation runt } y\text{-axeln}$$

Komplexa tal

Representation

$z = a + b\mathrm{i}$ Rektangulär form

$z = r(\cos v + \mathrm{i}\sin v)$ Polär form

$z = r\mathrm{e}^{\mathrm{i}v}$ Exponentiell form

Argument

$\arg z = v \qquad \tan v = \dfrac{b}{a}$

Absolutbelopp

$$|z| = r = \sqrt{a^2 + b^2}$$

Konjugat

$$\overline{z} = a - b\mathrm{i}$$

Räknelagar

$$z_1 \cdot z_2 = r_1 \cdot r_2(\cos(v_1 + v_2) + \mathrm{i}\sin(v_1 + v_2))$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(v_1 - v_2) + \mathrm{i}\sin(v_1 - v_2))$$

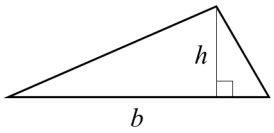
de Moivres formel

$$z^n = (r(\cos v + \mathrm{i}\sin v))^n = r^n(\cos nv + \mathrm{i}\sin nv)$$

Geometri

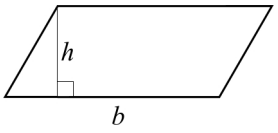
Triangel

$$A = \frac{bh}{2}$$



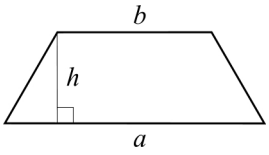
Parallellogram

$$A = bh$$



Parallelltrapets

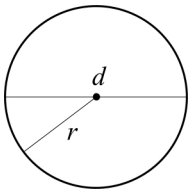
$$A = \frac{h(a+b)}{2}$$



Cirkel

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

$$O = 2\pi r = \pi d$$



Cirkelsektor

v mäts i grader

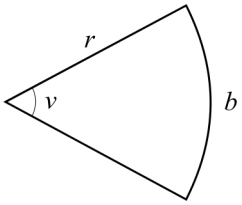
$$b = \frac{v}{360°} \cdot 2\pi r$$

$$A = \frac{v}{360°} \cdot \pi r^2 = \frac{br}{2}$$

v mäts i radianer

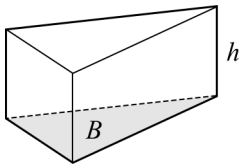
$$b = vr$$

$$A = \frac{vr^2}{2} = \frac{br}{2}$$



Prisma

$$V = Bh$$

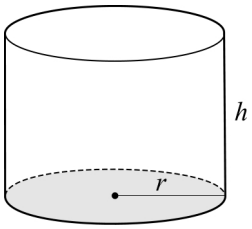


Cylinder

$$V = \pi r^2 h$$

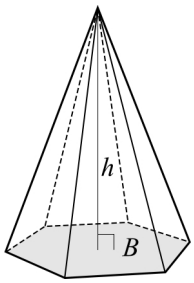
Mantelarea

$$A = 2\pi rh$$



Pyramid

$$V = \frac{Bh}{3}$$

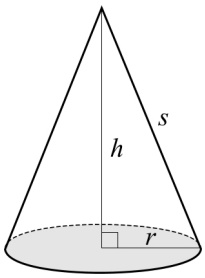


Kon

$$V = \frac{\pi r^2 h}{3}$$

Mantelarea

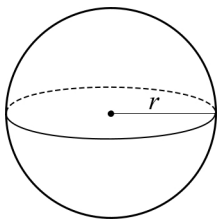
$$A = \pi rs$$



Klot

$$V = \frac{4\pi r^3}{3}$$

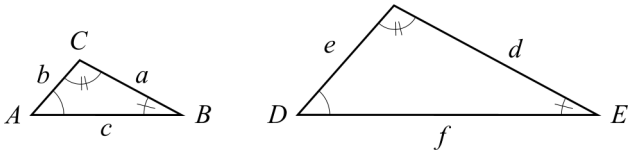
$$A = 4\pi r^2$$



Likformighet

Trianglarna ABC och DEF är likformiga

om $\frac{a}{d}=\frac{b}{e}=\frac{c}{f}$



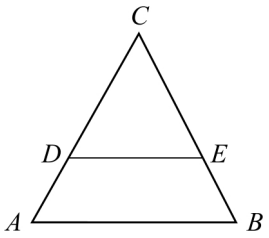
Skala Areaskalan = (Längdskalan)^2 Volymskalan = (Längdskalan)^3

Topptriangelsatsen

$\frac{DE}{AB}=\frac{CD}{AC}=\frac{CE}{BC}$

Transversalsatsen

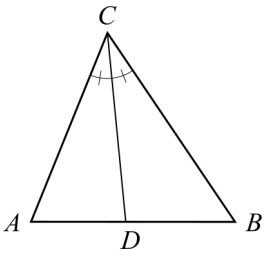
$\frac{CD}{AD}=\frac{CE}{BE}$



DE är parallell med AB

Bisektrissatsen

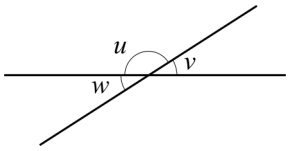
$\frac{AD}{BD}=\frac{AC}{BC}$



Vinklar

$u+v=180°$ Sidovinklar

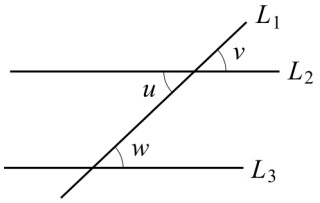
$w=v$ Vertikalvinklar



L_1 skär två parallella linjer L_2 och L_3

$v=w$ Likbelägna vinklar

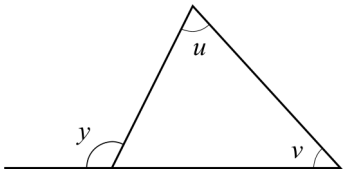
$u=w$ Alternatvinklar



Vinkelsumman S i en n-hörning: $S=(n-2)\cdot 180°$

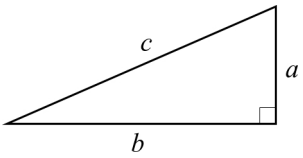
Yttervinkelsatsen

$y=u+v$



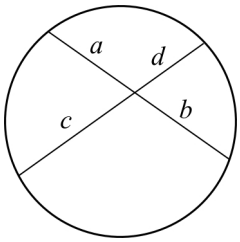
Pythagoras sats

$a^2+b^2=c^2$



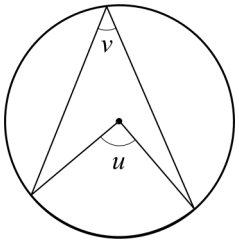
Kordasatsen

$ab=cd$



Randvinkelsatsen

$u=2v$



Avståndsformeln

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

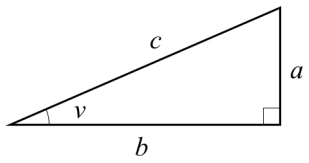
Mittpunktsformeln

$x_m=\frac{x_1+x_2}{2}$ och $y_m=\frac{y_1+y_2}{2}$

Trigonometri

Definitioner

Rätvinklig triangel

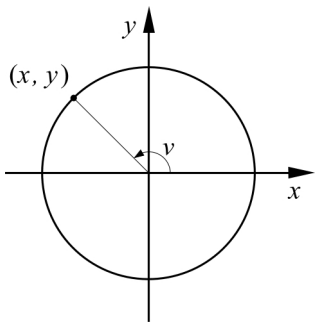


$$\sin v = \frac{a}{c}$$

$$\cos v = \frac{b}{c}$$

$$\tan v = \frac{a}{b}$$

Enhetscirkel



$$\sin v = y$$

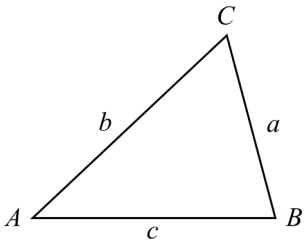
$$\cos v = x$$

$$\tan v = \frac{y}{x}$$

Sinussatsen $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Cosinussatsen $a^2 = b^2 + c^2 - 2bc\cos A$

Areasatsen $T = \dfrac{ab\sin C}{2}$



Trigonometriska formler

$$\sin^2 v + \cos^2 v = 1$$

$$\sin(u+v) = \sin u\cos v + \cos u\sin v$$

$$\sin(u-v) = \sin u\cos v - \cos u\sin v$$

$$\cos(u+v) = \cos u\cos v - \sin u\sin v$$

$$\cos(u-v) = \cos u\cos v + \sin u\sin v$$

$$\sin 2v = 2\sin v\cos v$$

$$\cos 2v = \begin{cases} \cos^2 v - \sin^2 v & (1) \\ 2\cos^2 v - 1 & (2) \\ 1 - 2\sin^2 v & (3) \end{cases}$$

$a\sin x + b\cos x = c\sin(x+v)$ där $c = \sqrt{a^2+b^2}$ och $\tan v = \dfrac{b}{a}$

Trigonometriska funktionsvärden

Vinkel v									
(grader)	0°	30°	45°	60°	90°	120°	135°	150°	180°
(radianer)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin v$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan v$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Ej def.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0